A fuzzy-based model for unbalanced bidding in construction

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Abstract
Existing unbalanced bidding models assume that quantities of work are certain and deterministic. However in reality some works such as soil and rock excavation cannot be estimated accurately before the work is completely done. In an unbalanced bid, the contractor must take the responsibility of uncertainties in quantities of work. It may be difficult to define probability distribution functions for these parameters due to lack of data and information. This paper presents a fuzzy linear programming (FLP) model of unbalanced bidding which assigns fuzzy numbers to the quantity of works. The model maximizes the present value of the profit as an objective function. Model is applied to a hypothetical case example and compares the results of fuzzy and deterministic models.

Keywords
unbalanced bidding, fuzzy modeling, fuzzy linear programming

1. Introduction
It is usually advantageous for a contractor to use unbalanced bidding strategies in order to benefit from the uneven distribution of markup among the project’s component items. The contractor has to decide on the unit prices of all the project’s activities so that the summation of the project’s component items equals to the tender price. This strategy leads contractor to price some items with a high markup and others with a low one in order to compensate.

Tendering relatively high unit prices on items scheduled for early completion and proportionately lower prices for other items is a type of unbalanced bidding. Well managed unbalanced bidding can help to obtain monies earlier for financing the later stages to improve cash flow of the project.

Contractors may provide their own estimate of the quantities for each items of the contract. If their estimations differ from those in proposals, another type of unbalanced bidding could be formulated by overpricing items expected to exceed the proposal estimates while underpricing the ones which appear to be overestimated. This strategy can help the contractor to submit a lower bid or improve the profitability of his tender. However, there can be substantial risk. Several models have been presented for unbalanced bidding. It is believed that Marvin Gates’ strategy (1959, 1967) was the first to comment on the practice of unbalanced bidding, Stark’s model (1968, 1972, 1974) advocated a linear programming solutions for unbalanced bidding, later Diekmann’s et al. (1982),

In practice some works such as soil and rock excavation can be measured but cannot be estimated accurately before the activity is completely done. These uncertainties in quantities can cause additional risks for the contractor. None of researchers has considered this uncertainty except Diekmann et al. (1982) who changed Stark’s original deterministic model by adding a probabilistic formulation to take into account risk of uncertainties in quantities of work. It is almost impossible to define probability distribution functions for these work items due to lack of data and information.

Since Bellman and Zadeh presented a decision making process in 1970 in a fuzzy environment, different (fuzzy linear programming) FLP models have emerged and evolved to incorporate the imprecision of goals and constraints in an (linear programming) LP model [Chanas and Kuchta (1994), Ramik and Rimanek (1985) and Tanaka and Asai (1984)]. This paper presents a (FLP) model to deal with the uncertainty of the parameters in an optimization model of unbalanced bidding. The paper changes Stark’s original deterministic model by assuming the quantity of works as fuzzy numbers which make the model a FLP model with fuzzy coefficients in objective function and crisp constraints.

2. Deterministic unbalanced bidding model, General structure

A linear programming model for unbalanced bidding may be defined as: [Stark,1974]

$$\text{Max} Z = \sum_{i=1}^{n} \sum_{j=1}^{t} d^j p_{ij} (X_{ij} - C_{ij})Q_i - \sum_{j=1}^{t} d^j F_j = \sum_{i=1}^{n} A_i Q_i (X_{ij} - C_{ij}) - F_0$$  \hspace{1cm} (1)

Subject to:

$$\sum_{i=1}^{n} P_i X_i = B$$  \hspace{1cm} (2)

$$L_i \leq X_{ij} \leq U_i$$  \hspace{1cm} (3)

$$X_{re} - X_{ec} \geq 0$$  \hspace{1cm} (4)

In which \( d = (1 + r)^{-1} \) = discount factor, \( r \) = interest rate, \( n \) = number of items, \( t \) = total number of periods; \( p_{ij} \) = percent of item \( i \) to be completed in the \( j \)th period, such that \( \sum_{j=1}^{t} p_{ij} = 1 \), \( X_i \) = the unit price of the \( i \)th item (decision variable), \( C_i \) = unit cost of the \( i \)th item, \( Q_i \) = quantity of item \( i \) (in the bidding documents), \( P_i \) = the quantity of the \( i \)th item measured by the owner, \( F_j \) = Fixed costs which are overhead and mobilization cost during period \( j \), \( A_i = \sum_{j=1}^{t} d^j p_{ij} \), \( F_o = \sum_{j=1}^{t} d^j F_j \) = present worth of the fixed cost.

The maximization of \( Z_o \) has restrictions, suppose the contractor wishes to submit a total bid, \( B \), the unit price of each item must be chosen in a way that their sum, when multiplied by the owner’s quantity estimates, equals the total bid amount, \( B \). This constraint is presented as Eq.2.

Eq.3 makes sure that the unit prices fall within specified bounds. This constraint ensures that the unit prices are relevant to the direct costs as a protection against quantity misestimates or to hide pricing policies.

In practice, certain unit prices cannot be more or less than the other; e.g., the unit price for rock excavation, \( X_{re} \), normally exceeds that of earth excavation, \( X_{ec} \) (Eq.3).
As contract payments are based on actual quantities of work, a fuzzy-based model might be more useful than the deterministic one suggested above.

### 3. FLP model with fuzzy quantities of work

Considering the problem discussed in previous section with fuzzy quantities of work, one may formulate the problem as

$$\text{Max} Z = \sum_{t=1}^{n} \sum_{j=1}^{m} d_{ij} p_{ij} (X_i - C_j) \tilde{Q}_{ij} - \sum_{j=1}^{m} d_{ij} F_j = \sum_{j=1}^{m} A_j \tilde{Q}_j (X_i - C_j) - F_0$$

Subject to: Eqs. (2, 3, 4 and 6) (5)

$$\tilde{Q} = (Q, Q, m, n)_{L-L}$$ (6)

Where the coefficient $\tilde{Q}$ defines fuzzy numbers of the type L-L. $Q$ and $Q$ refer to left and right borders of fuzzy number $\tilde{Q}$ corresponding to maximum reliability level (i.e., $\alpha=1$) and $m$ and $n$ are nonnegative real numbers (Fig.1).

![Figure 1: Typical membership function of a trapezoidal fuzzy number](image)

The problem defined by objective function Eq.(5) and constraints Eqs.(2)-(4) can be associated with a set of problems, which depend on a parameter $\alpha \in (0,1]$ as follows (Chanas and Kuchta 1994):

$$\text{Max} Z = \sum_{j=1}^{n} A_j \tilde{Q}_j^\alpha (X_i - C_j) - F_0$$

Subject to:
Eqs. (2)-(4) (7)

where the quantity of work $Q_j^\alpha$ in the objective function represent the intervals corresponding to the $\alpha$ cut of fuzzy number $\tilde{Q}$

After some manipulation employing fuzzy algebra, the FLP model of unbalanced bidding with symmetric triangular fuzzy numbers will be equivalent to [Chanas & Kuchta,1996]:
where $Z^L$ and $Z^R$=alternative crisp objective functions which represent the lower and upper bounds of total present value of profit for a certain $\alpha$ cut, respectively. In general the objective function can be defined as:

$$Z' = \sum [1 + (2t - 1)(1-\alpha)] Q A (X - c) - F_0$$

(9)

where $Z^L = Z'^{t=0}$ , $Z^R = Z'^{t=1}$ and $t=0.5$ results in the objective function of the nonfuzzy unbalanced bidding model.

The constrained method is employed for dealing with bi-objective programming. The model is solved for each objective function at different $\alpha$ cuts. Let’s define $X^L$ and $X^R$ as the sets of decision variables that individually optimize $Z^L$ and $Z^R$ at a certain $\alpha$ cut, respectively. Therefore each of the $Z^L$ and $Z^R$ will assume two different values for any $\alpha$ cut through the single objective solution to the equation (8) and the subjected constraints [Eqs.2-4]. Two different membership functions may now be constructed for $X^L$ and $X^R$. The plot of $Z^L$ and $Z^R$ associated with $X^L$ and different values of $\alpha$ cuts results in the first membership function. The second one results from plot of $Z^L$ and $Z^R$ associated with $X^R$ and different values of $\alpha$ cuts.

For this bi-objective problem nondominated solutions must be found. Therefore, mathematical presentation of the fuzzy model which trades off between maximum values of $Z^L$ and $Z^R$ using the linear membership functions $\mu(Z^R)$ and $\mu(Z^L)$ will result as (Karimi et al. 2007):

$$\text{MAX}=T$$

$$Z' (x) = \sum [(1+k(1-\alpha)) Q A (X - C_i)] - F_0 \geq Z_{min}^r + T(Z_{max}^r - Z_{min}^r)$$

$$Z' (x) = \sum [(1-k(1-\alpha)) Q A (X - C_i)] - F_0 \geq Z_{min}^l + T(Z_{max}^l - Z_{min}^l)$$

$$\sum_{i=1}^{n} P_i X_i = B$$

$$L_i \leq X_i \leq U_i$$

(10) $X_3 \geq X_2$

Where $0 \leq T \leq 1$ is the trade off between two objective functions which causes highest possible degree of membership for the two membership functions.

Plot of $Z^L$ and $Z^R$, obtained from solution to the model defined by Eq.10, for different values of $\alpha$ cut forms the traded-off membership shape of the objective function.

This results in a set of optimum solutions for certain $\alpha$ cuts. Since the decision maker needs to know the optimum crisp unit prices, those variables must be defuzzified. One of the most common defuzzifying operators is the centroid defuzzifier. In this approach the value of $T=T_G$ must be identified by solving the set of equation (10) for $\alpha=\alpha_G$ where point G is the centroid of the traded-off membership shape of the objective function. The value $T_G$ is inserted into Eq.(11), in order to obtain the optimum crisp values of the decision variables.
\[ MIN \left\{ \frac{\text{MAX} \left\{ Z^{G^T}_{\text{Max}Z^T} \cdot (Z^{G^T}_{\text{Max}Z^T} - (Z^{G^T}_{\text{Min}Z^T}) \right\}}{Z^{G^T}_{\text{Max}Z^T} - (Z^{G^T}_{\text{Min}Z^T})} - T \right\} \]

\[ \alpha = \alpha_G \]
\[ T = T_G \]
\[ (Z^{G^T})_{\text{praded-off}} = Z_G \]

\[ (Z^{G^T}_{\text{Max}Z^T}) = \sum (l + k(2G_l - 1)(1 - \alpha)Q(A(X_i^R - C_i)) - F_0) \]
\[ (Z^{G^T}_{\text{Min}Z^T}) = \sum (l + k(2G_l - 1)(1 - \alpha)Q(A(X_i^L - C_i)) - F_0) \]
\[ (Z^{G^T}_{\text{praded-off}}) = \sum (l + k(2G_l - 1)(1 - \alpha)Q(A(X_i - C_i)) - F_0) \]
\[ \sum_{i=1}^{n} P_i X_i = B \]
\[ L_i \leq X_i \leq U_i \]
\[ X_3 \geq X_2 \quad (11) \]

4. Model application

Consider the example which is solved by Diekmann et al. (1982) with probabilistic formulation. A bid for a foundation work is prepared. In general for a balanced bidding, the contractor determines the total job cost, adds an 8% markup and prorates the total amount among items of the proposal. The items and their quantities and preparation of a typical “balanced bid” are shown in Table 1 and Table 2, respectively.

<table>
<thead>
<tr>
<th>Item number</th>
<th>Description</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Clear and grub</td>
<td>lump sum</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Earth excavation</td>
<td>cubic yard</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>Rock excavation</td>
<td>cubic yard</td>
<td>400</td>
</tr>
<tr>
<td>4</td>
<td>Caissons, 24 in. diam, installed</td>
<td>lineal foot</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>Clean-up</td>
<td>lump sum</td>
<td>1</td>
</tr>
</tbody>
</table>

The contractor thinks that the quantities of earth and rock excavation might be different from those in proposal. In order to take uncertainties in quantities into account they are assumed as triangular fuzzy numbers.

<table>
<thead>
<tr>
<th>item number</th>
<th>unit cost, in dollars (1)</th>
<th>proposal quantity (2)</th>
<th>item cost (estimated) in dollars (3)</th>
<th>unit price in dollars (4)</th>
<th>item totals, in dollars (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8,000.00</td>
<td>1.00</td>
<td>8,000.00</td>
<td>10,080.00</td>
<td>10,080.00</td>
</tr>
<tr>
<td>2</td>
<td>20.00</td>
<td>1,000.00</td>
<td>20,000.00</td>
<td>25.20</td>
<td>25,200.00</td>
</tr>
<tr>
<td>3</td>
<td>50.00</td>
<td>200.00</td>
<td>10,000.00</td>
<td>63.00</td>
<td>12,600.00</td>
</tr>
<tr>
<td>4</td>
<td>45.00</td>
<td>400.00</td>
<td>18,000.00</td>
<td>56.70</td>
<td>22,680.00</td>
</tr>
</tbody>
</table>
Triangular fuzzy quantity values in the objective function with center values equal to the values of crisp ones is presented in table 1. For second and third items in table 1, 20 and 10 percent fuzziness are assumed, respectively.

The model was solved for all $\alpha$ cuts with maximizing one of the objective functions and the other objective function was obtained. The two membership shapes of the objective functions are achieved by plotting $Z_l^e$ and $Z_r^e$ for all $\alpha$ cuts while maximizing each objective function. In Fig.1 curves AB and AE are variations of $Z_l^e$ and $Z_r^e$ for different $\alpha$ cuts for decision variables which maximize $Z_l^e$. Similarly, if they are plotted based on the solution optimizing $Z_r^e$, the membership function corresponding to $X_l^e$ will be obtained (i.e., triangle $CAD$ in Fig. 1). Break points on each curve represent changes in structure of the optimum solution in that $\alpha$ cut.

![Figure: 1. Membership shape of the traded-off objective function](image)

By solving the set of equation (10), the traded off membership shape of the objective function (i.e., triangle $FAG$) and its centroid at $\alpha_G=0.33$ is determined which corresponds to $Z_G=5478.9$. The optimum decision variables as presented in Table 3 are achieved by substituting the model parameters and $T_G$ in set of Eqs. (11). The optimum solutions of both fuzzy and nonfuzzy problems are also compared in Table 3. The fuzzy model has succeeded to take into account all possible situations with only 0.02% reduction in profit.

**Table 3: results of unbalanced bidding with Fuzzy and Nonfuzzy quantities of work**

<table>
<thead>
<tr>
<th>variable</th>
<th>data ranges</th>
<th>Fuzzy method</th>
<th>nonfuzzy method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0-1</td>
<td>0.33</td>
<td>1</td>
</tr>
<tr>
<td>T</td>
<td>0-1</td>
<td>0.574</td>
<td>1</td>
</tr>
<tr>
<td>$z$</td>
<td>3828.418-7128.538</td>
<td>5478.985</td>
<td>5480</td>
</tr>
<tr>
<td>$q2$</td>
<td>800-1200</td>
<td>1003.618</td>
<td>1000</td>
</tr>
</tbody>
</table>
### 6. Conclusion

A FLP model of unbalanced bidding was presented to incorporate the uncertainty of those values and their consequences in the unbalanced bidding model while maximizing present value of the profit. The uncertainty in fuzzy quantities of work in the objective function and its effect on decision variables of the model was considered using the method presented by Chanas and Kuchta (1994).

A case example [Diekmann et al. (1982)], was solved and the results were compared with the crisp ones. In comparison to the LP method, the presented FLP method incorporates all possible situations which make it more realistic whereas the present value of the profit reduction was not significant. Employing this proposed model and its results, the contractors could have a better understanding of the effects of the unit prices on their overall earning.

### 7. References


