Optimization of uncertain construction time-cost trade-off problem

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Abstract
Time–cost optimization (TCO) may be defined as a process to identify suitable construction activities for speeding up, and for deciding “by how much” so as to attain the best possible savings in both time and cost. In reality due to different uncertainties, the actual cost and time of each option is not certainly known for the manager in advance. Therefore, total time and cost of project may differ significantly because of these uncertainties. In this paper, fuzzy logic theory is employed to consider affecting uncertainties in total time, direct and indirect cost of a construction project. A multi objective optimization algorithm based on genetic algorithm (GA), is applied to provide a trade-off between implementation time and total cost. Project manager can also have different non-dominated solutions or Pareto solutions which are dependent on his measure of accepted risk through applying α-cuts methods in fuzzy logic theory. The proposed model leads the decision maker to select the desirable Pareto front solution through acceptable value of α-cut.

Keywords
Time-cost, trade off, fuzzy theory, GA, Decision making

1. Introduction
Time and cost as two critical objectives of construction project management, are not independent but intricately related. Trade-offs between project duration and total cost are extensively discussed in the project scheduling literature because of its practical relevance. Time cost trade off problem is one of the highly important issues in project accomplishment and has been ever taken into consideration by project managers. It is generally realized that when project duration is compressed, the project will call for an increase in labor and more productive equipment, and require more demanding procurement and construction management, and then the cost will increase. Time–cost optimization may be defined as a process to identify suitable construction activities for speeding up, and for deciding “by how much” so as to attain the best possible savings in both time and cost. Since there is a hidden tradeoff relationship between project time and cost, it might be difficult to predict whether the total cost (i.e., the direct and indirect costs) would increase or decrease as a result of the schedule compression. Since different combinations of possible durations and costs for the activities can be associated with a project,
problem is which of these combinations are the best. Determining the best sets is the goal of time-cost optimization.

2. Literature reviews

A wide variety of heuristic procedures were used to solve the time–cost tradeoff problem (Fondahl 1961; Siemens 1971; Moselhi 1993). In general, these procedures provided rule-of-thumb guidelines for crashing activities with least costs but cannot guarantee optimality. Mathematical programming models constitute another group to tackle the problem. Reda and Carr (1989) applied mixed IP to solve the time–cost tradeoff between related activities. Liu et al. (1995) employed LP/IP hybrid method to locate the lower bound of the project time–cost relationship through linear programming and then find the exact solution by means of integer programming. Successful development and application of meta heuristic optimization algorithms for solving single objective optimization problems in recent years has attracted the attention of researchers to investigate the possibility of their application to solve multi objective optimization problems.

There exist numerous difficulties and complexities in applying meta heuristic algorithms to solve multi objective problems and several researchers have engaged in appropriate use of these methods during past 2 decades. In this regards, different versions of GAs have successfully been applied to optimize time cost problem (Feng et al. 1997, Li and Love 1997, Hegazy 1999, Zheng et al. 2004).

Uncertainties in the problem have received less attention due its complexity. Therefore, most of the researches have been focused on deterministic problems. In real construction projects however, time and cost of activities may face significant changes due to existing uncertainties such as inflation, economical and social stresses, execution errors of contractor, design errors, natural events such as climate changes and etc. Therefore, total time and cost of project may differ significantly because of these uncertainties.

In this paper, a new approach has been investigated in solving time-cost trade off problem. Fuzzy logic theory is employed to consider affecting uncertainties in time, total direct and indirect cost of a construction project. To obtain appropriate solutions, genetic algorithm has been employed as an optimizer where uncertainties are considered through fuzzy logic representation. Project manager can also have different non-dominated solutions or pareto solutions which are dependent on his measure of accepted risk through applying $\alpha$ cuts methods in fuzzy logic theory. A case study through which considerable conclusions is drawn is finally investigated.

3. Basis Concept

3.1. Fuzzy Sets

Fuzzy set theory has been applied to many areas which need to manage uncertain and vague data. Such areas include approximate reasoning, decision making, optimization and control. Fuzzy numbers are a special kind of fuzzy set, which are normal and convex. Although these numbers can be described by using many special methods and shapes, triangular and trapezoidal shapes are widely best used for solving practical applications. The $\alpha$ cut is a commonly used method to connect the principles of fuzzy sets with a collection of crisp sets, which can in turn be fed into most of the existing systems. The $\alpha$ -cut level set or $\alpha$ -level cut of $A$ is the set:

$$A^\alpha = \{x, \mu_A(x) \geq \alpha | x \in X\} \quad \forall \alpha \in [0,1]$$

where $X$=range of possible values; and $\mu_A(x)$ =membership function taking values from $[0,1]$, specifying to what degree $x$ belongs to the fuzzy set $A$. 
The α represents the degree of risk that the managers is prepared to take. Since the value of α could severely influence the non dominated solutions, its choice should be carefully considered by decision makers. As a result, a general survey which aims at identifying the relationship between the value of α (i.e., 0 to 1) and the corresponding risk level (i.e., no risk to full risk) by collecting the risk attitudes of managers would be indispensable.

3.2. Genetic Algorithm (GA)

Over the last few years, scientists, engineers and economists have extensively used genetic algorithms (GA), to solve optimization problems involving single objective functions. During last few years several researchers have extended GA to solve multi objective problems. The basic operation of a genetic algorithm is simple. First a population of possible solutions to a problem is developed. Then, the better solutions are recombined with each other to form some new solutions for the next generation. In each generation, the fitness of every individual in the population is evaluated, multiple individuals are stochastically selected from the current population (based on their fitness), and modified (crossover and possibly mutation) to form a new population. The new population is then used in the next iteration of the algorithm. Finally the new solutions are used to replace the poorer of the original solutions and the process is repeated. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population.

4. Suggested Model

In reality due to different uncertainties involved time and cost of each option is not a crisp value. In another words, the actual time and cost of each option is not certainly known for the manager in advance. However, after project execution, they will be known. To apply time and cost of each option may be well considered employing fuzzy set theory. So, the number presented for each activity time and cost is a fuzzy one.

A triangular fuzzy number \((X = (x_1, x_2, x_3))\) may be assigned for the time or cost of that activity. Defining \(c_1, c_2\) and \(c_3\) as optimistic, pessimistic and the most probable time or cost for an activity, respectively. To solve time-cost trade off problem, some options can be chosen for implementation of each activity. For example, if there exist 7 activities and 5 options for each activity, then \(5^7\) sets of solutions will exist. Therefore, genetic algorithm is applied to obtain optimal solutions of the problem. Length of chromosome or number of genes equals to the number of activities and value of each gene is the option considered for fulfillment of the corresponding activity. New terminology will be introduced, along with step by step procedure.

Step1- A real value which is smaller than maximum number of each activity options is randomly chosen for each gene. Values of each gene are then read and choose option of each activity is assigned. So, a time and a cost value which is in the form of triangular fuzzy number are defined. When reading all the genes value is terminated, fuzzy time and cost value are assigned to all the project activities. Then reading the genes of next chromosome starts so that the first generation is finished.

Step2- Project critical path for each chromosome is defined using network bases and equations, considering the governing concepts of the network and predecessor activities. Summation of fuzzy times of a set of options in each path may be defined with a fuzzy number. Fuzzy mathematical equations are employed to add the fuzzy time number. Assume that \(X_1\) and \(X_2\) are two fuzzy number and their \(\alpha\) cuts are presented as \(X_1^\alpha_1\) and \(X_2^\alpha_2\). The sum of these two numbers would be as follows:

\[
(X_1 + X_2)_\alpha = [x_1^- + x_2^-, x_1^+ + x_2^+]
\]

\(\forall \alpha \in [0,1]\)
In order to address the critical path, the total time of each path associated fuzzy times for different values of $\alpha$ cut must be compared and ranked. To compare fuzzy times, they should be transformed to a crisp value through application of center of gravity defuzzifier. Therefore, if total fuzzy time ($\tilde{X}$) is covered by membership function $A$, the center of gravity defuzzifier defines point $X^*$ as the center of region which is covered by $A$. For $\alpha = 1$, value of $X^*$ will represent the total time of a path in fully crisp environment.

$$\int X\mu_A(x)dx$$

The time of project termination for each path can be compared with other using fuzzy numbers comparison method based on center of gravity defuzzifier. This process leads to obtain project termination time.

Step 3- Total direct cost of project is calculated. It is necessary, in this situation, to add costs of project activities to each other according to the options determined for them. Fuzzy mathematical equation (Eq.2) is employed to add the fuzzy cost number.

Step 4- Model has the ability to easily calculate the total project cost. Total project cost will be obtained by multiplying daily cost (indirect cost) by project execution time and adding this product to the relevant direct cost. If the indirect cost is fuzzy number, fuzzy mathematical equations will be employed to multiply this number because time and cost are fuzzy numbers. Assume that $\tilde{X}1$ and $\tilde{X}2$ are fuzzy numbers, the multiplication of these two numbers would be as follows [13]:

$$\tilde{X} = \tilde{X}1 \times \tilde{X}2 = [\min(x1^-_\alpha \times x2^-_\alpha, x1^-_\alpha \times x2^+_\alpha, x1^+_\alpha \times x2^-_\alpha, x1^+_\alpha \times x2^+_\alpha),$$

$$[\max(x1^-_\alpha \times x2^-_\alpha, x1^-_\alpha \times x2^+_\alpha, x1^+_\alpha \times x2^-_\alpha, x1^+_\alpha \times x2^+_\alpha)]$$

After multiplying indirect cost to time in a fuzzy manner using above mentioned equations and adding the indirect cost to direct cost in a fuzzy manner (Eq.2), total cost which is a triangular fuzzy number will be obtained. Therefore, there is a fuzzy value for total project time and cost.

Step 5- An order pair including time and cost of project termination is formed for each chromosome or solution of the project. The chromosomes than which no chromosome is formed having lower both time and cost, are named non-dominated chromosome. Pareto solutions of each population are the non-dominate chromosomes. In order to find dominated and non-dominated solutions, it is necessary to make a comparison between the times and the costs of chromosomes. In this situation, fuzzy numbers comparison and ranking methods is applied based on center of gravity defuzzifier. The cost and time of project termination for each alternative solution (chromosome) can be compared with other (chromosomes) binarily using fuzzy numbers comparison method.

Step 6- Euclidian distance ($d_E$) of each individual (i) from each non-dominate individual (NI) is calculated according following equation:

$$d_E(i, ni) = \sqrt{(C^*_i - C^*_ni)^2 + (T^*_i - T^*_ni)^2}$$

Where $C^*_\text{max} = \text{defuzzy maximum cost of population}$; $C^*_\text{min} = \text{defuzzy minimum cost of population}$; $C^*_i = \text{defuzzy cost of individual i}$; $C^*_n = \text{defuzzy cost of non-dominate individual ni}$; $T^*_\text{max} = \text{defuzzy maximum time of population}$; $T^*_\text{min} = \text{defuzzy minimum time of population}$; $T^*_i = \text{defuzzy time of individual i}$; $T^*_n = \text{defuzzy time of non-dominate individual ni}$.

Step 7- Minimum of Euclidian distances is considered as fitness function for each chromosome.

$$f_i = \text{Min}(d_E(i, ni) \text{ for all ni})$$
Basically, non-dominate solutions have fitness as equal to zero and the others belong more or less fitness values in proportion to their distance from non-dominate solution.

Step 8- Because every chromosome which is closer to non-dominated solutions is better, reverse of fitness function values is considered as reproduction rate of each chromosome.

\[
P_i = \frac{1}{f_i + 0.0001}
\]

To avoid non-dominate solutions fitness values, i.e. zero, to be situated in denominator of fraction, a very small number is allocated to their fitness values and then population are produced based on reversed fitness function.

Step 9- To produce population of next generation, non-dominate solutions is kept as optimal solutions for next generation and so these chromosomes are produced again. In order to obtain other chromosomes of the next generation, some of reproduced chromosomes are chosen to be used in crossover and mutation processes and so reproduced offspring constitute other population of next generation.

Step 10- Repeat 2-9 until the pareto front remains the same after a pre-specified number of iterations. The principal behind the above approach is that for each individual within a generation- the closer to non-dominate solution it is, the more fit. Therefore there will be a natural tendency for the new population to move toward the pareto solution. As new populations are produced, a pareto front will tend to move toward the coordinate axes. When the pareto front can no longer move closer to the coordinate axis, the solutions is found. Because, each non-dominate individual will be kept for the next generation, so that the pareto front always more toward to the coordinate axes.

Advantage of this model in comparison with similar ones lies on the method that, uncertainties in project time and cost are demonstrated, aggregated and interpreted. Triangular fuzzy values are assumed for any individual option with total time and cost being presented as a fuzzy number. So, when project manager chooses his optimum solution, he would face a total time and cost and corresponding membership function ahead that considerably help him to make appropriate decisions based on his own level of risk acceptance. The project manager may apply his own risk acceptance level to obtain a new pareto front with new non-dominated solutions using α cuts property. When he dose not want to risk, he would assign zero for α and therefore, times and costs of project activities, direct cost and total time and cost obtained for the project may be subject to high range of changes. When project manager wants to fully risk (i.e. 100 percent), he sets α equal to 1 and so, times and costs of project activities, direct cost and total time and cost would be transformed from a fuzzy value to a crisp one and problem would enter into a crisp space from a fuzzy one. In this situation, uncertainties in time and cost estimation would be practically ignored. Determination of α (i.e. accepting different risk percentage), would lead to different pareto solutions.

5. Case study

To demonstrate the concept and test the performance of the proposed model, a simple case example was adopted from Mr. Zheng (2004). It consists of 7 activities with different possible options. The time and cost data reported by Zheng et al (2004) were assumed associate with the most probable condition with membership of 1; while for the minimum and maximum time and cost for any option time and cost values were assumed to form the triangular membership functions. Adopted and assumed values for time and cost of options along with other required data for 7 activity project are presented in table1. Assumed value for indirect cost is (410, 500, 720) dollars.
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The example has been solved for different values of $\alpha$ cuts and results have shown in Table2.

<table>
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<tr>
<th>$\alpha$ cut number</th>
<th>Pareto solution of different $\alpha$ cut</th>
<th>Cost</th>
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Assuming $\alpha = 1$, disregards uncertainties and solve the model in an absolutely crisp space. In this case, it is expected to obtain the same solution reported by previous researcher for a fully crisp problem. In fact, solution to the model was exactly the same solution reported by Zheng et al (2004) for a fully deterministic solution space. To demonstrate the pareto front in a time-cost coordinate system, fuzzy time and cost will be transformed to a crisp value through application of center of gravity defuzzifier. Fuzzy times and costs related to $\alpha = 0$, $\alpha = 0.4$, $\alpha = 0.8$ and $\alpha = 1$ have been transformed to crisp values by center of gravity defuzzifier, and pareto fronts in a time-cost coordinate system are shown in figure 1.
As is clear, when project manager changes his risk acceptance he would face a different pareto front that considerably help him to make appropriate decisions. The lower the $\alpha$, the higher the total cost for any given time has been resulted. In another words, pareto front moves upward as it value of $\alpha$ approaches to zero. This issue means that for the lower risk, the higher time and cost would be accrued for project execution. In this case total time and cost will have smaller range of changes. However, assuming bigger values for $\alpha$ associates with the higher risk acceptance, which results in the lower time and cost with quite the larger range of changes. This is the major benefit of this model application. The example can be solved for different values of $\alpha$ cuts for direct and indirect cost. It means that, project manager may apply different risk acceptance level for direct cost and indirect cost separately.

6-Conclusion

Time cost trade off problem is one of the highly important issues in project accomplishment and has been ever taken into consideration by project managers. In a real construction project, time and cost of each activity and daily cost change as a result of many uncertainties such as inflation, economical and social stresses, execution errors of contractor, design errors, natural events. Genetic algorithm is used for extraction the pareto front. The model adopts fuzzy sets to simulate the degree of uncertainty of the input data. The incorporation of fuzzy sets theory in time cost trade off problems is therefore a sensible step to emulate the decision-making process of human experts based on a set of uncertain or incomplete data. Unlike the traditional models which focus primarily on the deterministic time cost trade off problem, this model has the ability to adapt to deterministic and uncertain environment by changing the value of $\alpha$. The project manager can apply his own risk acceptance level to obtain a new pareto front with new non-dominated solutions using $\alpha$ cuts property. For the lower risk, the higher time and cost would be accrued.
for project execution. Project manager can apply different risk acceptance level for direct cost and indirect cost separately.

7. References


