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USE OF CONFORMAL MAPPING TECHNIQUE TO THEORETICALLY VISUALIZE THE FLOW AROUND AN AIRFOIL GENERATED FROM A LIFTING CYLINDER IN A UNIFORM AERODYNAMIC FLOW FOR FLUSH PORT AIR DATA SYSTEM

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ABSTRACT: *Conformal Mapping is one of the various techniques by virtue of which we can develop an airfoil from basic elementary flows. Flow around an Airfoil to calculate C_P Variations have always been an important/basic part of Aerodynamic Study. Flush System or Flush Port Air Data System (FADS) is typically used in Stealth Technology to calculate AOA of an aircraft by using corresponding pressure ports.*

SYMBOLS USED

ψ = Streamfunction
ϕ = Velocity Potential
AOA = AOA
w.r.t = with respect to
$\Delta P = P_{Downstream} - P_{Upstream}$
$\Delta C_P = C_{P(Downstream)} - C_{P(Upstream)}$
α = Value of AOA
θ = Angular variation over the body surface, CW rotation opposing the direction of flow is taken positive
r = Radial distance from the center of the body
SLC = St&ard Sea Level Conditions
P=2116psf, T=518.69^{0}R, Speed of Sound, a=1116 ft/s
Γ = Circulation,
Λ= Source Strength in ft^2/s
κ = Doublet Strength in ft^3/s
M_∞ = Free Stream Mach Number
U_∞= Free Stream Velocity
$C_{p(Corrected)} = C_p /[(1-M_\infty^2)^{1/2}+\{M_\infty^2 / (1+(1-M_\infty^2)^{1/2})\}C_p /2]$
[(Karman-Tsien Compressibility Correction]
R_o, θ_o, b are parameters in Conformal Mapping of Airfoil

1. INTRODUCTION

Conformal Mapping is a technique used in mathematical physics to transform a complicated shape into simpler one so that the mathematical treatment can be given it easily. In this technique if we know the relation w=f(z), then region Z in the z plane has a corresponding region W in the w plane & every point in one plane has a corresponding point in the other plane. This transformation is analytic and it preserves angle. This theory can be used to extend the application of potential flow theory to practical aerodynamics. Standard potential flow theory begins with an ideal flow to show that lift on a body is proportional to the circulation about a closed path encompassing an object. Potential flows start with flows over cylinders since the mathematics is more tractable. However, to use potential flow theory on usable airfoils one must rely on conformal mapping to show a relation between realistic airfoil shapes & the knowledge gained from flow about cylinders. Hence it can be used in the design of an airfoil section. By assuming such a flow about a body, flow remains attach with the surface of the body & zero streamline defines the profile of the body. Therefore exploiting this property a streamline can be expressed to conformal transformation.

Actually this paper is a small part of author's another paper "A Step towards Stealth Technology – Pressure Differential Angle of Attack Measuring System" in which the author has suggested the location of pressure ports to install Flush Port Air Data System (FADS) typically used in Stealth Technology.

2. MATHEMATICAL DESCRIPTION

The basic flows used in potential flow theory such as uniform flow, source, sink, doublet & vortex, can all be represented using complex numbers e.g. if a complex number *w* with both real & imaginary parts represents a potential flow (2.1). Different Elementary Flow in complex plane are

$$w(z) = \phi + i\psi \quad \rightarrow (2.1)$$

Uniform flow: $$w(z) = U_\infty z = \phi + i\psi = U_\infty(x+iy) = U_\infty x + iU_\infty y = U_\infty r\cos\theta + iU_\infty r\sin\theta^{[1,4]} \rightarrow (2.2)$$

Source flow: $$w(z) = \frac{\Lambda}{2\pi}\ln(z) = \phi + i\psi = \frac{\Lambda}{2\pi}\ln(re^{i\theta}) = \frac{\Lambda}{2\pi}(\ln(r) + i\theta) = \frac{\Lambda}{2\pi}\ln(r) + i\frac{\Lambda}{2\pi}\theta^{[1,4]}. \quad \rightarrow (2.3)$$

Vortex flow: $w(z)=i\frac{\Gamma}{2\pi}\ln(z)=\phi+i\psi=i\frac{\Gamma}{2\pi}\ln(re^{i\theta})=i\frac{\Gamma}{2\pi}(\ln(r)+i\theta)=-\frac{\Gamma}{2\pi}\theta+i\frac{\Gamma}{2\pi}\ln(r)$ *[1,4]* .→ (2.4)

Doublet flow: $w(z)=\frac{k}{2\pi}\frac{1}{z}=\phi+i\psi=\frac{k}{2\pi}\frac{1}{re^{i\theta}}=\frac{k}{2\pi}\left(\frac{1}{r}e^{-i\theta}\right)=\frac{k}{2\pi}\frac{1}{r}(\cos\theta-i\sin\theta)$ *[1,4]* → (2.5)

In complex terms the flow past a cylinder with lift is written as in (2.6):

$$w(z)=U_\infty\left(z+\frac{R^2}{z}\right)+i\frac{\Gamma}{2\pi}\ln(z) \quad \text{[1,4]} \qquad \rightarrow (2.6)$$

When a potential flow is represented in complex form, the velocity components can be found using (2.7)

$$\frac{dw}{dz}=u-iv \quad \text{[1,4]} \qquad \rightarrow (2.7)$$

Flow around an airfoil is closely related to the combination of circulation & parallel flow around a circular cylinder. It is possible to transform the airfoil profile analytically into a circle, and the flow around the circle can define the desired flow around the airfoil. Instead of mapping directly from the w plane to the z plane a transformation is performed in stages using 4 planes. In this a case the functional relationship w=f(z) includes a set of successive transformation equations. |For aerodynamics applications the Joukowski transform is the most commonly used function; given in (2.8) which is actually a complex potential of flow past a circular cylinder without lift.

$$w(z)=z+\frac{b^2}{z} \quad \text{[4]} \qquad \rightarrow (2.8)$$

Here, *b* is a constant. Graphically, a conformal mapping transforms a complex plane in *z* ($z = x+iy$) into a complex plane in a new variable *w* ($w = \xi+i\zeta$). A uniform flow in the *z* plane is transformed into an equivalent form in the *w* plane using a transform of the form $w = f(z)$. Similarly a circle drawn in the *z* plane, $z = be^{i\theta}$ transforms into a flat plate. Using Joukowski Transform, (2.8) we get (2.9). A circle of radius *b* is mapped into a straight line in the *w* plane entirely on the real axis between -2*b* & 2*b*.

$$w=be^{i\theta}+\frac{b^2}{be^{i\theta}}=be^{i\theta}+be^{-i\theta}=2b\cos(\theta)+i0 \quad \text{[,4]} \qquad \rightarrow (2.9)$$

Now a uniform flow had been drawn over the circle, the transform would have mapped that flow into the flow over a flat plate in the *w* plane. If the circle originally had a radius slightly larger than the transform constant *b*, $z = ae^{i\theta}$, with $a > b$, the circle would have formed an ellipse instead of the flat plate (2.10) and (2.11).

$$w=z+\frac{b^2}{z}=ae^{i\theta}+\frac{b^2}{ae^{i\theta}}=\left(a+\frac{b^2}{a}\right)\cos(\theta)+i\left(a-\frac{b^2}{a}\right)\sin(\theta)=x+iy \quad \text{[4]} \qquad \rightarrow (2.10)$$

$$\frac{x^2}{\left(a+\frac{b^2}{a}\right)^2}+\frac{y^2}{\left(a-\frac{b^2}{a}\right)^2}=1 \quad \text{[4]} \qquad \rightarrow (2.11)$$

In this paper a circle is made to slightly offset from the origin along the negative real axis, to obtain symmetric Joukowski airfoil. The equation of the offset circle is $z = ae^{i\theta}-eb$ where the constant *e* is a small number as shown in Fig 2.1. If the cylinder is displaced slightly along the complex axis as well, one obtains a cambered airfoil shape as shown in Fig 2.2.

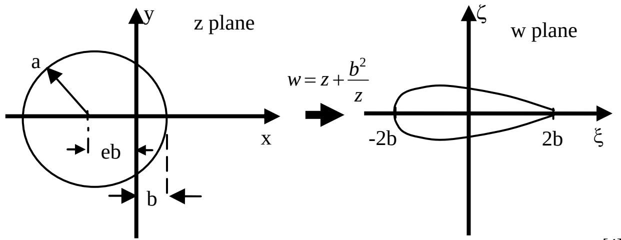

Fig 2.1 Transformation of Cylinder into Symmetrical Airfoil[4]

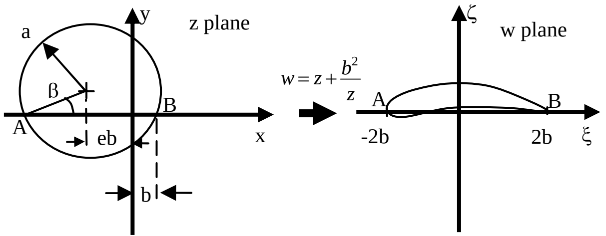

Fig 2.2 Transformation of Cylinder into a Cambered Airfoil[4]

Points A & *B* are the intercepts of the displaced circle on the real axis. The angle β is the angle formed by the line joining the point *A* (or *B)* & the origin with the real axis. Due to the lifting flow about the original circle the Joukowski transformation generates a lifting flow about the Joukowski airfoil. The stagnation points on the cylinder map to stagnation points that are not realistic. The only means of making a realistic flow is to impose the Kutta condition. This is done by adjusting the value of vorticity strength Γ, so that the stagnation points on the cylinder reside at the cylinder's intercepts of the real axis. In this case, when the cylinder is transformed, one stagnation point is forced to the trailing edge. To evaluate the lift, the circulation is needed & therefore the velocity field. The velocity field in each plane is related to each other through the chain rule of differentiation. If the lifting flow about the cylinder is defined as function *Q* where *Q = Q(z)* in the *z* plane & *Q = Q(w)* in *w* plane, the velocities in each plane are $V_z=\partial Q/\partial z$ & $V_w=\partial Q/\partial w$. By chain rule we get

$$\frac{\partial Q}{\partial z}=\frac{\partial Q}{\partial w}\frac{\partial w}{\partial z}\Rightarrow V_z=V_w\frac{\partial w}{\partial z}\ [4] \qquad \rightarrow (2.12)$$

From Joukowski transformation we have; $\frac{\partial w}{\partial z}=\frac{z^2-b^2}{z^2}$ *[4]* → (2.13)

Clearly, the velocity field very close to the cylinder & its transformed counterpart are dissimilar. The circulations must be the same in both planes i.e. $\rho V_\infty \Gamma_{\text{cylinder}}=.\rho V_\infty \Gamma_{\text{jowkouski}}$. The appropriate vortex strength to impose the Kutta is then determined. Consider the lifting flow about a cylinder. The velocity in the θ direction is found to be Eq 2.14. This velocity is zero at stagnation point on cylinder. At these points $\theta=-\beta$.

$$V_\theta=-\left(2V_\infty\sin(\theta)+\frac{\Gamma}{2\pi R}\right)\ [4] \qquad \rightarrow (2.14)$$

$$\Gamma=4\pi V_\infty R\sin(\beta)\ [4] \qquad \rightarrow (2.15)$$

Now if the field is rotated by α to simulate then, $\Gamma=4\pi V_\infty R\sin(\beta+\alpha)$ *[4]* → (2.16)

Also by simple mathematics we can get the value of β as,

$$\beta=\sin^{-1}\left(\frac{R_o}{R}\sin\theta_o\right)\&\, b=-R_o\cos\theta_o+\sqrt{R^2-R_o^2\sin^2\theta_o} \qquad \rightarrow (2.17)$$

The potential flow past a Joukowski airfoil at an AOA is obtained with the aid of successive transformations. Eq 2.18 represents the complex potential for a flow past a lifting cylinder of radius R with clockwise circulation. Equation 2.19 rotates the flow pattern through angle α. Eq 2.20 shifts z_2 in the z_3 plane the center O of the circle from the origin of the co-ordinate axis at a distance m & at an angle δ with respect to x_3 axes & finally Eq 2.21 transforms the circle of radius R into Jowkowski Airfoil. Using different values of R_o & θ_o different geometries are obtained as shown in Fig 2.3. The velocity field around the Jowkowski Airfoil is obtained using Eq 2.22.

$W = U_\infty(z_i + R^2/z_1) + i\Gamma/2\pi \ln(z_1/R)$ →(2.18)
$z_2 = z_1e^{i\alpha}$ → (2.19)
$z_3 = z_2 + R_o\exp(i\theta_o)$ → (2.20)
$z = z_3 + b^2/z_3$. → (2.21)

$$\frac{\partial w}{\partial z}=\frac{\partial w}{\partial z_1}\frac{\partial z_1}{\partial z_2}\frac{\partial z_2}{\partial z_3}\frac{\partial z_3}{\partial z} \qquad \rightarrow (2.22)$$

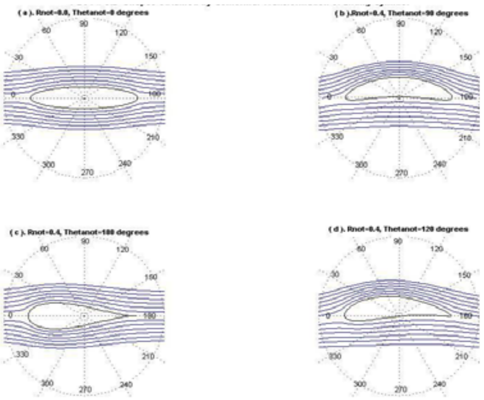

Fig 2.3 Formation of different geometries using different values of R_o & θ_o in MatLab

To simulate these flow over the airfoil, a doublet of strength κ=2513 ft^2/s was chosen & then it wa superimposed with uniform flow at an angle α of velocity U_∞=100 ft/s & a circulation of strength 1000ft^2/s. SLC were used. The stream function equation used is given as

$$\psi(r,\theta)=U_\infty r\sin(\theta-\alpha)\left(1-\frac{R^2}{r^2}\right)+2RU_\infty\sin(\alpha+\beta)\ln\left(\frac{r}{R}\right) \qquad \rightarrow (2.23)$$

Varying r & ϑ at ψ=0 at different AOA, the values of V_r & V_θ are calculated on the cylinder which are again transformed into V_r & V_θ on the airfoil using the given relations. With the help of these values of V, C_p values are then evaluated using the relation. $C_p = 1 - (V/U_\infty)^2$, are shown in Fig 2.4 for 20% thick cambered airfoil, Fig 2.5 for 20% thick symmetrical airfoil, Fig 2.6 for 6% thick cambered airfoil & in Fig 2.7 for 6% thick symmetrical airfoil. These four types of airfoils are generated using different values of R_o & θ_o. They are, 0.2t/c Cambered Airfoil,$R_o=0.4$, $\vartheta_o=120^0$; 0.2t/c Symmetric Airfoil $R_o=0.3, \vartheta_o=180^0$; 0.06t/c Cambered Airfoil $R_o=0.15\,\vartheta_o=120^0$; 0.06t/c Symmetric Airfoil $R_o=0.06\,\vartheta_o=180^0$. The C_p data obtained from these airfoils was then compared with the Experimental Data of NACA Airfoils similar to these airfoils.

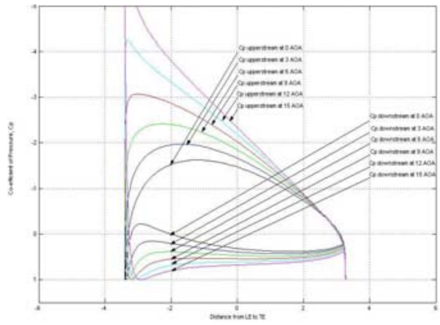

Fig 2.4 C_P variation on 20% thick cambered airfoil

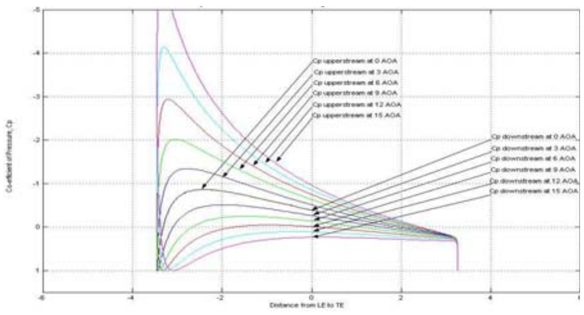

Fig 2.5 C_P variation on 20% thick symmetrical airfoil

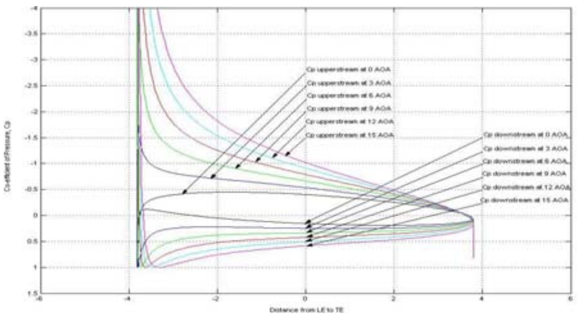

Fig 2.6 C_P variation on 6% thick symmetrical airfoil

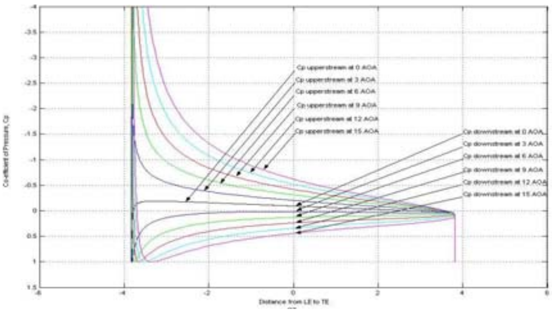

Fig 2.7 C_P variation on 6% thick cambered airfoil

CONCLUSIONS

i. In some problems of aerodynamics of small speeds the air can be considered as an ideal noncompressible fluid. In the process of the calculation of streaming around the aerofoil of a large span the aerofoil can be considered as a long cylinder & the flow of an air. In this case to calculate the velocity field of air it is possible to use the Conformal Mapping Technique which is very easy to formulate. Also we can use this technique to design airfoils as per our requirement especially to reduce RCS Signature.

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