NED UNIVERSITY JOURNAL OF RESEARCH


# GEOMETRIC MEAN DERIVATIVE-BASED OPEN NEWTON-COTES QUADRATURE RULES

Author(s): Sara Mahesar[1], Muhammad Mujtaba Shaikh[2], Kashif Memon[3], Muhammad Saleem Chandio[4]

Volume: **XXII**
No: **4**
Pages: **248-275**
Date: **December 2025**

**DOI:** https://doi.org/10.35453/NEDJR-ASCN013.R6

**Abstract:**
A novel family of open Newton-Cotes formulas, termed GMDONC, is proposed and designed to enhance the accuracy of evaluating definite integrals through numerical integrators that are polynomial interpolatory in nature. By incorporating the geometric mean in the even-order derivatives of the integrand within the interval [$a$, $b$], the GMDONC methods exhibit a significant two-order accuracy improvement over traditional ONC approaches. Theorems on degree of precision, order of accuracy, and error terms are derived validating the theoretical advancements. Computational analyses confirm the superior performance of GMDONC through assessments of computational cost, CPU time, and error reductions across various integrals. Comparative evaluations with Gauss-Legendre methods highlight the effectiveness of GMDONC in handling integrals with diverse characteristics, including regular, oscillatory, periodic, and singular integrals. The proposed rules demonstrate computational and time efficiency in the global context compared to the existing polynomial ONC and Gauss-Legendre rules with the same number of functional nodes. The scope of the present improvement is restricted only to the context of polynomial interpolatory quadrature, not in the sense of spline/semi-interpolatory quadrature.

[1] Assistant Professor, Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Pakistan., Ph. +923413922005, Email: sara.mahesar@faculty.muet.edu.pk
[2] Associate Professor, Department of Basic Sciences and Related Studies, Mehran University of Engineering and Technology, Jamshoro, Pakistan, Ph. +923332617602, Email: mujtaba.shaikh@faculty.muet.edu.pk
[3] Assistant Professor, Institute of Mathematics and Computer Sciences, University of Sindh, Jamshoro, Pakistan, Ph. +923332791141 Email: memonkashif.84@gmail.com
[4] Former Professor, Institute of Mathematics and Computer Sciences, University of Sindh, Jamshoro, Pakistan, Ph. +923322060869 Email: mschandio@hotmail.com

**Keywords:** Open Newton-Cotes rules, geometric mean, precision, accuracy, computing cost.

**For full paper, contact:**
**Prof Muhammad Imran Aslam**
Editor-in-Chief, NED University Journal of Research
Ph: +92 (21) 99261261-8 Ext:2670; Fax: +92 (21) 99261255
Email: NED-Journal@neduet.edu.pk
Website: http://www.neduet.edu.pk/NED-Journal